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# A heavy little Higgs and a light $Z'$ under the radar

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## Abstract

The original littlest Higgs model with universal fermion couplings is found to be consistent with precision electroweak data but is strongly constrained by Tevatron limits on the predicted centi-weak  $Z'$  boson. A possible signal observed by CDF at 240 GeV is consistent with the predicted  $Z'$ , and a region below 150 GeV is largely unconstrained by collider data. LHC searches for narrow dilepton resonances below 500 GeV will have sufficient sensitivity to discover the  $Z'$  boson or to exclude the model over most of the range allowed by the electroweak fits.

## Introduction

Little Higgs models address the fine tuning problem posed by quadratically divergent one loop corrections to the Higgs boson mass in the SM (Standard Model) by identifying the Higgs as a pseudo-Nambu-Goldstone boson which only acquires a cutoff sensitive mass at two loop order. Pioneering studies of the original  $SU(5)/SO(5)$  littlest Higgs model[1] found that constraints from precision EW (electroweak) data[2, 3, 4, 5] and collider limits on the predicted  $Z'$  boson[6] force the model into a region where fine-tuning re-emerges, engendering many variants of the original model.[7] Here we present EW fits of the original model that are consistent with the precision data and in which the Higgs mass is not fine-tuned. Good fits, with  $\chi^2$  values below the SM fit and fine tuning above 10% (and often of order one), occur for values of the  $SU(5)$  breaking condensate  $f$  between 0.5 and 2.7 TeV. The best of these fits are at  $f = O(1)$  TeV, as originally envisioned in [1], while unexpectedly favoring large values of the Higgs boson mass, from  $\sim 0.3 - 1$  TeV. The model then also removes the tension between the EW data and the LEP II lower limit on the Higgs mass, which is especially acute if the  $A_{FB}^b$  anomaly is due to underestimated systematic error.[8]

A signature prediction of the fits is a light  $Z'$  boson below  $\simeq 500$  GeV and possibly as light as  $O(100)$  GeV, with centi-weak coupling to quarks and leptons. CDF[10] and D0[11] limits currently provide the strongest constraints, excluding much of the region allowed by the EW fits. An excess at 240 GeV in the  $e^+e^-$  mass spectrum observed by CDF[10] is consistent with the  $Z'$  predicted by the EW fits. The excess is nominally  $3.8\sigma$ , with 0.6% probability ( $2.5\sigma$ ) to be due to a chance fluctuation anywhere in the 150-1000 GeV mass range. If confirmed as a  $Z'$  boson, it would correspond in the LH (littlest Higgs) model to a symmetry breaking scale  $f \simeq 1.5$  TeV, and would provide a good fit to the EW data. The CDF and D0 studies have comparable sensitivity, since D0 considered a larger data sample while CDF had a larger acceptance, and the CDF excess is outside the D0 allowed region. Future Tevatron and LHC data will soon determine if the excess is a fluctuation or a real signal. The model can be tuned to further suppress  $Z'$  production, but without a physical basis from the UV completion it would be strongly disfavored unless a signal emerges at or near the present limits.

Following [3] we assume universal fermion charge assignments for the two  $U(1)$  gauge groups embedded in the global  $SO(5)$ : the first two SM families have the same  $U(1)_i$  charges as the third family, fixed by gauge invariance of the top quark Yukawa interaction specified in the original model[1] and the absence of mixed  $SU(2)_L - U(1)_i$  anomalies. The results differ from earlier studies[3, 5] chiefly because the EW fits are performed with complete scans of both the SM and LH parameters, possible thanks to currently available computing capability. Earlier studies fixed the SM parameters at their SM best fit values and/or did not scan on all LH parameters. We find that the LH best fit typically occurs at different values

of the SM parameters than the SM best fit (especially  $m_H$ ) and that important cancellations emerge if all LH parameters are scanned. Current data is more restrictive than the data used in earlier studies — in addition to more precise measurements of the top and  $W$  masses, low energy data[12, 13] and Tevatron limits on  $Z'$  production now impose stronger constraints. ZFITTER[14] is used for the SM corrections, and experimental correlations are included.[15]

In the next section we discuss the methodology of the EW fits and summarize the results. We then discuss the light centi-weak  $Z'$  boson that is predicted by the fits, including the upper bounds from the Tevatron and the fits that result if the excess at 240 GeV seen by CDF is attributed to the  $Z'$ . This is followed by a discussion of the future limits that can be reached at the LHC. We next discuss the extent to which the parameters of the model are themselves fine tuned, concluding that the principal source of tuning is the constraint imposed on the  $U(1)$  mixing angle by the Tevatron (and eventually LHC) bounds on  $Z'$  production. We conclude with a brief discussion of the results.

## Electroweak Fits

The global  $SU(5)$  contains a gauged  $SU(2)_1 \times SU(2)_2 \times U(1)_1 \times U(1)_2$  subgroup with coupling constants  $g_1, g_2, g'_1, g'_2$ . The breaking to  $SO(5)$  with condensate  $f$  gives masses to a combination of the  $SU(2)_i$  and  $U(1)_i$  gauge bosons. The orthogonal  $SU(2) \times U(1)$  is unbroken and the would-be Higgs boson is at this stage part of a massless Nambu-Goldstone boson doublet. The unbroken  $SU(2) \times U(1)$  is identified with the EW  $SU(2) \times U(1)$  and is subsequently broken by a Higgs boson vev (vacuum expectation value),  $v = 247$  GeV, induced by the one loop effective potential — for details see [1] and [2].

The salient features for the EW fit are (1) changes in  $Z$  boson interactions from  $Z - Z'$  mixing and (2) custodial  $SU(2)$  breaking from three sources: a triplet Higgs boson vev, the shift in  $m_Z$  due to  $Z - Z'$  mixing, and mixing between the left chirality  $t_L$  quark and its  $t'_L$  partner. We scan the usual SM parameters,  $\Delta\alpha^{(5)}$ ,  $\alpha_S$ ,  $m_t$ , and  $m_H$ , and the LH parameters which affect the fit: the  $SU(5)$  breaking scale  $f$ , the triplet Higgs vev  $v'$ , the sine of the  $t_L - t'_L$  mixing angle  $s_L$ , and the cosines of the  $SU(2)_i$  and  $U(1)_i$  mixing angles  $c$  and  $c'$ , related to the SM EW couplings by  $g = sg_1 = cg_2$  and  $g' = s'g'_1 = c'g'_2$ .

The universal fermion  $U(1)$  charge assignments are parameterized as  $y_1 = (1 - \eta')y_{SM}$  and  $y_2 = \eta'y_{SM}$ . Gauge invariance of the Yukawa interaction proposed in [1] then requires[3] that  $\eta' = 2/5$ , and the correction to the  $Z$  coupling for fermion  $f$  with SM coupling  $g_f = t_{3f} - s_W^2 q_f$  is

$$\delta g_f = \frac{v^2}{2f^2} \{t_{3f} [c^2(1 - 2c^2) + 5(c'^2 - \eta')(1 - 2c'^2)] - 5q_f(c'^2 - \eta')(1 - 2c'^2)\} \quad (1)$$

where  $t_{3f}$  and  $q_f$  are the weak isospin and charge of fermion  $f$ ,  $s_W^2 = \sin^2\theta_W$ , and  $\eta' = 2/5$

follows from the universal charge assignment. Corrections to the low energy parameters are

$$s_*^2(0) = s_W^2 \left\{ 1 - \frac{v^2}{2f^2} \left[ c^2 + 5(c'^2 - \eta')(1 - 2\eta') \left( 1 - \frac{1}{s_W^2} \right) \right] \right\} \quad (2)$$

and

$$\delta\rho_* = \frac{5}{4} \frac{v^2}{f^2} (1 - 2\eta')^2 - 4 \frac{v'^2}{v^2} \quad (3)$$

These results are consistent with [2, 3].<sup>1</sup>

The fits are performed subject to three conditions. First, requiring  $|v'| < |v^2/4f|$  ensures positivity of the triplet Higgs mass. Second, since the coefficient  $a$  of the quadratically divergent term in the one loop gauge boson effective potential is expected to be of order one, we require  $1/5 < a < 5$ , where  $a$  is determined by<sup>2</sup>

$$a = \frac{m_H^2}{4m_Z^2} \frac{c^2 c'^2}{s_W^2 c^2 + c_W^2 c'^2} \frac{1}{1 + |4v'f/v^2|}. \quad (4)$$

Third, following the earlier studies[1, 2, 3, 5] we consider the residual fine tuning from the top partner that cuts off the quadratically divergent top quark contribution to  $m_H$  and is the most important source of the SM little hierarchy problem.<sup>3</sup> We require this residual fine tuning to be no less than 10%,

$$\delta_{FT} = \frac{m_H^2}{(3m_t^2 m_t'^2 / 2\pi^2 v^2) \ln(4\pi f / m_{t'})} > 0.1, \quad (5)$$

where  $m_{t'}^2 = m_t^2 f / (s_L v - s_L^2 f)$ . Following [2, 3] we also restrict  $\theta$  and  $\theta'$  to  $s, c, s', c' \geq 0.1$  to keep the gauge coupling constants from becoming unreasonably large.

The 95% CL contour in the  $f - c'$  plane is shown in figure 1. The dashed line is the trajectory of the best fit. As in [3, 5] the contour is defined with respect to the SM best fit, although with a more restrictive criterion: we require  $\Delta\chi^2 < 5.99$  corresponding to 95% CL for two degrees of freedom ( $f$  and  $c'^2$ ), compared to  $\Delta\chi^2 < 7.8$  and  $\Delta\chi^2 < 6.6$  in [3] and [5] respectively. The global best fit with  $\chi^2 = 17.3$  is at  $f = 1.1$  TeV, 1.3  $\chi^2$  units below the SM best fit with  $\chi^2 = 18.6$ . As seen in figure 2 the  $\chi^2$  distribution as a function of  $f$  is extremely flat, varying by less than one  $\chi^2$  unit for  $f$  between 0.5 and 2.7 TeV. The upper limit at  $f = 3.5$  TeV is a consequence of the fine tuning constraint. The fits prefer large values of the Higgs boson mass, well above the 114 GeV LEP II lower limit. The  $\chi^2$  distribution is also very flat as a function of  $m_H$ , as can be seen in figure 3 for  $f = 1.1$  TeV, where  $\chi^2$  varies by no more than 0.2 units between  $m_H = 300$  GeV and  $m_H = 1$  TeV.

<sup>1</sup>Sign errors in eq. (3.10) of [2] do not propagate to the the appendix of [2] which we have verified.

<sup>2</sup>The potential eq. (4.16) of [2] reverses  $g_1 \leftrightarrow g_2$  and  $g'_1 \leftrightarrow g'_2$  relative to eq.(4.7) of [1]; we follow [1].

<sup>3</sup>For additional discussions of fine tuning in the LH model see [16, 17].

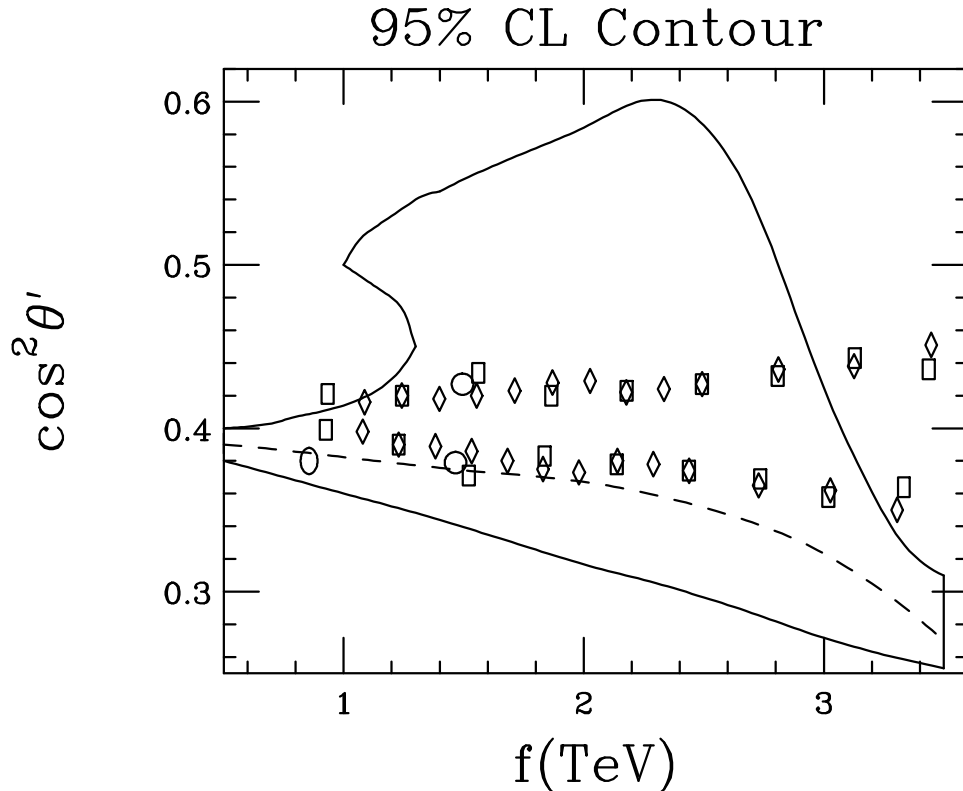


Figure 1: 95% CL contour for EW fits satisfying boundary conditions. The dashed line marks the best fit. Diamonds and boxes are upper and lower limits on  $c'^2$  obtained from the D0 and CDF limits on  $Z'$  production, and the two circles correspond to the CDF excess at 240 GeV. The ellipse corresponds to a  $Z'$  boson at 140 GeV that would be unobservable at LEP II, as discussed in the text.

These results are quite different from the earlier studies. In [3] fits with  $f = 1$  TeV are at the limit defined by  $\Delta\chi^2 = 7.8$ , hence  $9\chi^2$  units above the value obtained here, nor do they satisfy the fine tuning constraint. Those fits only improve as  $f$  increases, as the effects of the model begin to decouple. In contrast the best fits presented here are at  $f \simeq 1$  TeV and the upper limit on  $f$  is set by the fine tuning constraint. The difference is principally the result of scanning on the SM parameters, especially  $m_H$ , and on a more thorough scan over the LH parameters including the  $t - t'$  mixing angle  $s_L$ .

While the LH fit has more free parameters than the SM, the discovery of a  $Z'$  boson in the EW allowed region would determine the parameters  $f$  and  $c'$ , and the resulting LH fit would have a comparable confidence level to the SM. Because of the nature of the SM fit, the results obtained here are as good as it gets for any BSM model that does not explicitly address the  $3.2\sigma$   $A_{LR} - A_{FB}^b$  discrepancy with flavor-specific new physics, since all other data agree as well or better than chance with the SM.[8] The large pull of  $A_{FB}^b$  is entirely

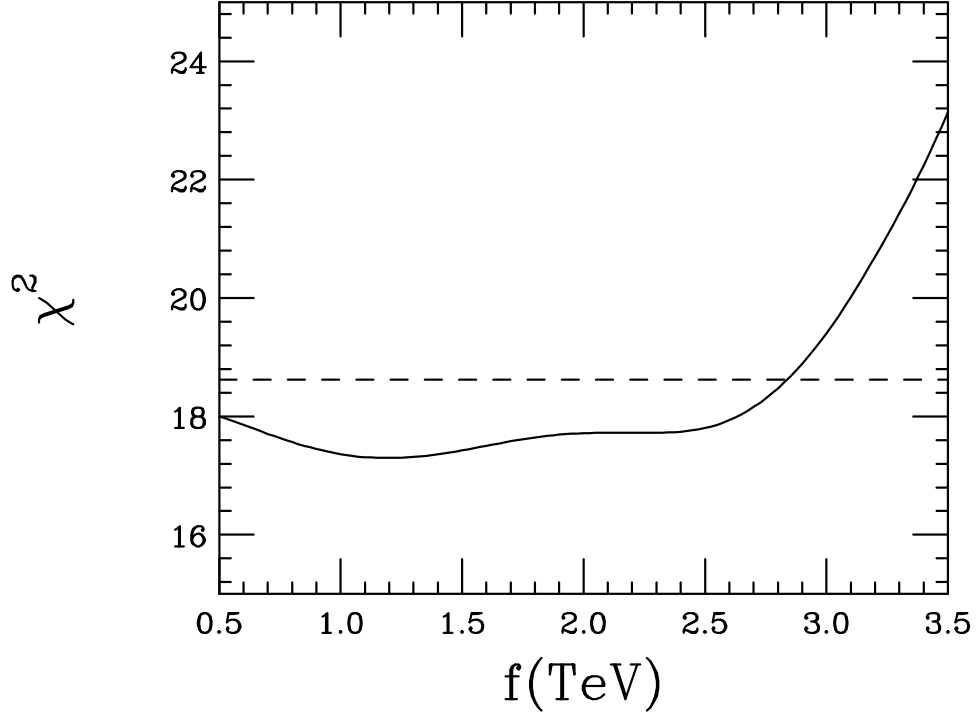


Figure 2:  $\chi^2$  as a function of  $f$  for the LH model. The dashed line indicates the  $\chi^2$  value of the SM best fit.

responsible for the marginal confidence level of the SM fit, as can be seen by comparing the SM fits in tables 2 and 3.

The  $b$  and  $c$  quark asymmetry measurements,  $A_{FB}^b$  and  $A_{FB}^c$ , have large QCD corrections that must be merged with the experimental cuts using hadronic Monte Carlos, giving rise to a systematic uncertainty that is difficult to estimate reliably.[9] If they are excluded the resulting SM fit is robust, with  $\chi^2$  confidence level increasing from  $CL(18.6, 13) = 0.14$  for the full data set (table 2) to  $CL(8.3, 11) = 0.69$  for the reduced set (table 3), but the central value for the Higgs mass decreases from 89 GeV, with 24% probability to be in the LEP II allowed region above 114 GeV, to 58 GeV, with only 4% probability for the allowed region.[8, 9] The LH model raises the predicted value of the Higgs mass for the reduced data set well above 114 GeV while maintaining the robust quality of the fit to the EW data. The best fit occurs at  $f = 1.4$  TeV and  $m_H = 520$  GeV with  $\chi^2 = 8.0$ . With discovery of a compatible  $Z'$  this would imply a 53% confidence level,  $CL(8.0, 9) = 0.53$ . For  $c' = 0.38$  and  $f = 1.47$  TeV, corresponding to the CDF excess at 240 GeV, the best fit has  $\chi^2 = 8.46$  and a confidence level of 0.49. The  $\chi^2$  of the best fit as a function of  $f$  and  $m_H$  is again quite flat as a function of  $m_H$ . For both data sets the  $\chi^2$  minimum is nearly independent of  $m_H$  (for large enough  $m_H$ ) because shifts in  $m_H$  are compensated by shifts in the LH parameters, especially  $s_L$  and  $v'$ .

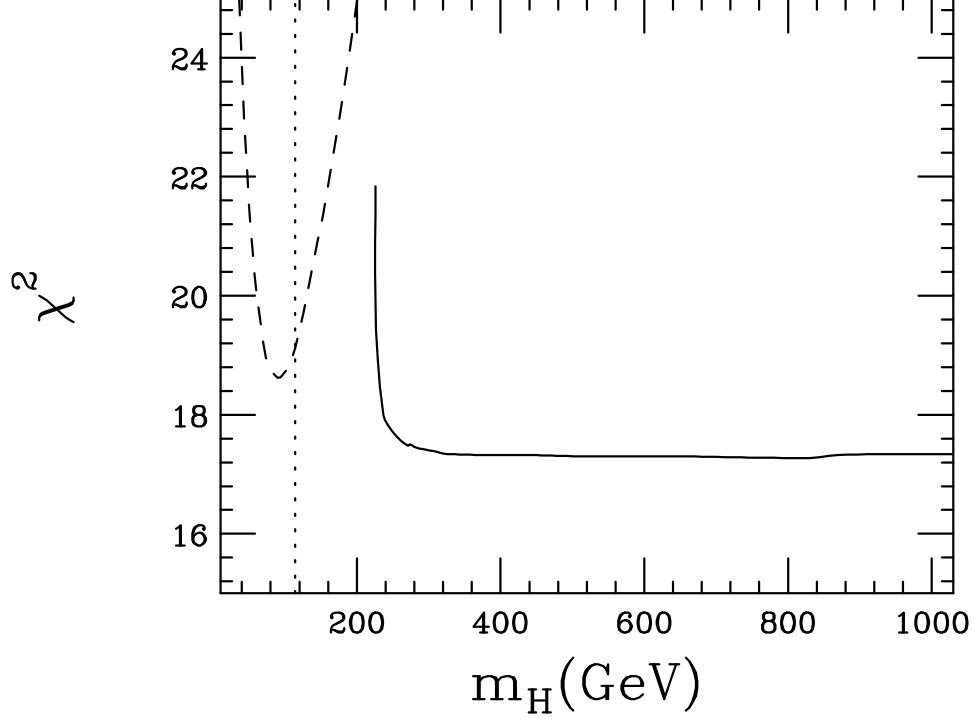


Figure 3:  $\chi^2$  distributions as a function of  $m_H$  for the LH model with  $f = 1.1$  TeV (solid line) subject to the three boundary conditions and for the SM (dashed line). The dotted line is the LEP II lower bound on  $m_H$ .

### The Centi-Weak $Z'$ Boson:

A characteristic prediction emerging from the fits is a light, narrow  $Z'_Y$  boson, between  $\sim 100$  and  $500$  GeV, coupled to SM hypercharge  $Y$ . The EW fits favor values of  $c'$  near  $\sqrt{\eta'} = \sqrt{0.4}$  which suppresses the strength of the coupling and reduces the effect of  $Z - Z'$  mixing on EW observables, as can be seen in equation (1). The mass is determined by the LH model parameters  $f$  and  $c'$ ,

$$m_{Z'} = \frac{s_W}{\sqrt{5}s'c'} \frac{f}{v} m_Z, \quad (6)$$

implying a light  $Z'$  boson, because of the factor  $1/\sqrt{5}$  and especially because the fits favor values of  $c'$  that maximize the factor  $s'c'$  in the denominator. Neglecting (for the moment)  $Z - Z'$  mixing, which is of order  $v^2/f^2$ , the  $Z'$ -fermion interaction is

$$\mathcal{L}_{Z'ff} = g_{Z'} \bar{f} \gamma_\mu \not{Z}'_\mu f \quad (7)$$

with  $y_f = q_f - t_{3f}$ , where  $g_{Z'}$  is related to the SM  $Z$  coupling  $g_Z = g/\cos\theta_W$  by

$$r_{Z'} \equiv \frac{g_{Z'}}{g_Z} = \frac{s_W(c'^2 - \eta')}{s'c'}. \quad (8)$$

However, because  $g_{Z'}$  is suppressed for  $c'^2$  near  $\eta' = 0.4$ , the small admixture of the SM  $Z_0$  boson in the  $Z'$  mass eigenstate can have a significant effect on the interaction of the  $Z'$ . With  $Z' \simeq Z_Y - \theta_{Z-Z'} Z_0$  the  $Z - Z'$  mixing angle at leading order is

$$\theta_{Z-Z'} = \frac{s_W(1 - 2c'^2)}{2s'c'} \frac{m_Z^2}{m_{Z'}^2}. \quad (9)$$

Including the effect of  $Z - Z'$  mixing on  $Z' f f$  interactions we replace equation (7) with

$$\mathcal{L}_{Z' f f} = g_Z \bar{f} g'_{f Z'} Z' f \quad (10)$$

where

$$g'_f = r_{Z'} y_f - \theta_{Z-Z'} (t_{3f} - s_W^2 q_f). \quad (11)$$

In particular, the  $Z'$  eigenstate then has an appreciable branching ratio to  $W^+ W^-$  when  $r_{Z'}$  is small.

Using equations (6) and (8-11) the  $Z'$  mass and couplings are determined in terms of  $f$  and  $c'$ . The cross section for  $Z'$  production as a function of  $m_{Z'}$  is then determined by the two parameters,  $f$  and  $c'$ . We compute the cross sections<sup>4</sup> with  $K$  factor  $K = 1.3$  and compare the results with the limits on narrow  $Z'$  production from CDF[10] and D0[11]. An upper limit on  $\sigma_{Z'} BR(e^+ e^-)$  at mass  $m_{Z'}$  implies upper and lower limits on  $c'^2$  at corresponding values of  $f$ , while a  $Z'$  discovery would determine  $f$  and  $c'^2$  up to a twofold ambiguity. Solving numerically we obtain the upper and lower limits on  $c'^2$  as a function of  $f$  that are shown in figure 1.

Viewing the CDF excess at 240 GeV as illustrative of a possible signal,<sup>5</sup> we estimate  $\sigma_{Z'} BR(e^+ e^-) \simeq 42$  fb from the data.<sup>6</sup> The corresponding values of  $f$  and  $c'^2$  are plotted as circles in figure 1 at (1.47 TeV, 0.38) and (1.50 TeV, 0.43). The solution at  $f = 1.47$  TeV is preferred by  $\Delta\chi^2 = 4$ , and the properties of the  $Z'$  boson for this choice are displayed in table 1. Because  $r_{Z'} = g_{Z'}/g_Z = 0.021$  and  $\theta_{Z-Z'} = 0.017$  are both small the  $Z'$  is extremely narrow, with a width of 11 MeV. Because  $r_{Z'}$  and  $\theta_{Z-Z'}$  are comparable in magnitude the

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<sup>4</sup>Collider cross sections are computed with Madgraph v4[18].

<sup>5</sup>Figure 1 of [10] compares the data to a fit assuming a  $Z'$  at 241.3 GeV with negligible intrinsic width. The fit predicts events principally in two bins between 230 and 250 GeV, while most of the excess is in the upper bin. However, with only  $\sim 30$  excess events no conclusion can be drawn from the shape of the distribution.

<sup>6</sup>The quoted net signal efficiency[10] at  $m_{e^+e^-} = 150$  GeV is  $\epsilon_{\text{TOT}} = 0.27$ . The net efficiency at 240 GeV increases in proportion to the acceptance of the CDF fiducial region, to  $\epsilon_{\text{TOT}} = 0.32$ , since trigger and other instrumental efficiencies for electrons in the fiducial region vary slowly between 150 and 240 GeV. Taking the interval  $m_{e^+e^-} = 240 \text{ GeV} \pm 2\sigma_{m_{e^+e^-}}$  where the CDF resolution at 240 GeV is  $\sigma_{m_{e^+e^-}} = 5.4$  GeV, we find 32 signal and 70 background events from figure 1 of [10], reproducing the quoted  $3.8\sigma$  nominal significance. The total signal cross section for  $2.5 \text{ fb}^{-1}$  is then  $\simeq 42$  fb.



$m_{Z'}$	240 GeV
$\sigma_{Z'} BR(e^+e^-)$	$\sim 42$ fb
$f$	1.47 TeV
$c'^2$	0.38
$r_{Z'} = g_{Z'}/g_Z$	0.021
$\theta_{Z-Z'}$	0.017
$\Gamma_{Z'}$	15 MeV
$BR(e^+e^-)$	0.10
$BR(\bar{\nu}_e \nu_e)$	0.0004
$BR(\bar{b}b)$	0.067
$BR(\bar{c}c)$	0.10
$BR(W^+W^-)$	0.29

Table 1: Properties of the hypothetical  $Z'$  boson based on the CDF  $e^+e^-$  excess at 240 GeV.

effect of  $Z'$  mixing on the properties of the  $Z'$  is significant. The  $e^+e^-$  branching ratio is 10%, reduced by  $Z - Z'$  mixing from the  $\sim 15\%$  branching ratio of a  $Z'$  boson coupled to hypercharge. There is a substantial 29% branching ratio to  $W^+W^-$  which is entirely due to the SM  $Z_0$  component of the  $Z'$  mass eigenstate. The  $e^+e^-$  branching ratio and  $W^+W^-$  decay can be used to distinguish the LH  $Z'$  boson from other narrow  $Z'$  bosons that couple predominantly to hypercharge, e.g., by kinetic mixing[19] or by the Stueckelberg mechanism.[20]

The best fit with  $Z'$  parameters corresponding to the 240 GeV CDF excess, as in table 1, has  $\chi^2 = 17.4$ , which is 1.2 units below the SM best fit. Varying  $m_H$ ,  $s_L$ , and  $v'$  there is a range of fits with similar  $\chi^2$  values, with  $m_H$  going from 270 GeV to 1 TeV and  $\delta_{FT}$  from 0.1 to 1.3. One of these, with  $m_H = 820$  GeV and  $\delta_{FT} = 0.9$ , is shown alongside the SM best fit in table 2. For this fit the masses of the top partner, triplet Higgs, and heavy  $W$  are 2.2, 7.2, and 2.1 TeV respectively. However these masses are not well determined since other fits with comparable  $\chi^2$  values predict different masses.

Because the Higgs triplet and the top partner both effect the EW fit predominantly via the oblique parameter  $T$ , the fit sees  $v'$  and  $s_L$  as a single degree of freedom.<sup>7</sup> Since a  $Z'$  discovery would determine  $f$  and  $c'$ , the resulting LH fit would effectively have two more independent parameters than the SM fit: the  $SU(2)$  mixing angle  $c$  and the oblique parameter  $T$  determined in a correlated way by  $v'$  and  $s_L$ . The LH fit to the full data set in

<sup>7</sup>However we vary  $v'$  and  $s_L$  separately to verify the boundary conditions, equations (4-5).

	Experiment	SM	Pull	LH	Pull
$A_{LR}$	0.1513 (21)	0.1480	1.6	0.1472	1.9
$A_{FB}^l$	0.01714 (95)	0.01644	0.7	0.01626	0.9
$A_{e,\tau}$	0.1465 (33)	0.1480	-0.5	0.1472	-0.2
$A_{FB}^b$	0.0992 (16)	0.1038	-2.9	0.1032	-2.5
$A_{FB}^c$	0.0707 (35)	0.0742	-1.0	0.0737	-0.9
$\Gamma_Z$	2495.2 (23)	2495.7	-0.2	2496.6	-0.6
$R_\ell$	20.767 (25)	20.739	1.1	20.741	1.0
$\sigma_h$	41.540 (37)	41.481	1.6	41.478	1.7
$R_b$	0.21629 (66)	0.21582	0.7	0.21561	1.0
$R_c$	0.1721 (30)	0.1722	-0.04	0.1723	-0.07
$A_b$	0.923 (20)	0.935	-0.6	0.935	-0.6
$A_c$	0.670 (27)	0.668	0.07	0.668	0.09
$m_W$	80.399 (23)	80.378	0.9	80.393	0.3
$A_{PV}$	$-131 (17) \cdot 10^{-9}$	$-156 \cdot 10^{-9}$	1.4	$-154 \cdot 10^{-9}$	1.3
$Q_W(Cs)$	-73.16 (.35)	-73.14	-0.06	-73.33	0.5
$\Delta\alpha^{(5)}(m_Z)$	0.02758 (35)	0.02768	-0.3	0.2761	-0.09
$m_t$	173.3 (1.1)	173.3	0.02	173.3	0.02
$\alpha_S(m_Z)$		0.1180		0.1198	
$m_H$		89		820	
$c$				0.24	
$v'$ (GeV)				1.0	
$s_L$				0.11	
$\chi^2/\text{dof}$		18.6/13		17.7/11	
$\text{CL}(\chi^2)$		0.135		0.09	
$m_H[90\%]$ (GeV)		51 — 152		270 — 1000	
$\text{CL}(m_H > 114 \text{ GeV})$		0.24		1	

Table 2: The SM best fit and an LH model fit with  $f, c'^2 = 1.47 \text{ TeV}, 0.38$  corresponding to the CDF excess at 240 GeV.

table 2 then has 11 degrees of freedom with confidence level  $CL(17.7, 11) = 0.09$ , comparable to the SM fit with  $CL(18.6, 13) = 0.135$ . For the reduced data set shown in table 3 with the hadronic asymmetries  $A_{FB}^b$  and  $A_{FB}^c$  omitted, the best LH fits have a robust confidence level,  $CL(8.6, 9) = 0.48$ , as does the SM fit with  $CL(8.3, 9) = 0.69$ . However, unlike the SM fit which predicts a 58 GeV Higgs boson with only 4% probability to be in the LEP II allowed region above 114 GeV, the LH fit has a shallow  $\chi^2$  minimum at  $m_H = 560$  GeV and is consistent with values between 270 GeV and 1 TeV.

## Future Prospects

Future Tevatron and LHC data will determine if the excess at 240 GeV is a real signal or, if not, can tighten the limits on  $c'^2$  to the point of implausibility unless motivated by UV completion of the model. As seen in figure 1 the difference between the upper and lower limits on  $c'^2$  from the current CDF and D0 searches ranges from 0.02 to 0.09 for  $m_{Z'}$  from 150 to 500 GeV. The limit scales with the integrated luminosity like  $\simeq L^{-\frac{1}{4}}$ . With  $L = 10 \text{ fb}^{-1}$  the CDF and D0 limits could tighten by 30% and 15% respectively.

To illustrate the sensitivity of the LHC for the LH  $Z'$  boson we consider  $m_{Z'} = 240$  GeV and  $c'^2 = 0.38$ , corresponding to the CDF excess. We require  $p_T > 25$  GeV and  $|\eta| < 2.4$  for  $e^+$  and  $e^-$ , and assume 65% efficiency within the fiducial region, as already achieved by ATLAS in an early study of the  $Z$  boson[21]. We parameterize the  $e^+e^-$  fractional mass resolution,  $\hat{\sigma}_m = \sigma_m/m$ , by  $d_m = \hat{\sigma}_m/0.02$ , since  $\hat{\sigma}_m \simeq 2\%$  for CDF at 240 GeV, a figure that will eventually be surpassed by ATLAS and CMS. The signal region is defined as  $m_{Z'} \pm 2\sigma_m$ . At  $\sqrt{s} = 7$  TeV the CDF  $3.8\sigma$  excess would then have a significance of  $9\sigma \cdot \sqrt{(L/d_m)}$  with  $L$  expressed in  $\text{fb}^{-1}$ . The current  $40 \text{ pb}^{-1}$  data sample is inconclusive since it corresponds to only  $1.8\sigma$  for  $d_m = 1$ , while the  $\gtrsim 1 \text{ fb}^{-1}$  sample projected for the coming year can decisively exclude or confirm the predicted signal. For the LHC at 7 TeV with  $L = d = 1$  the expected 95% CL limit on production of the 240 GeV LH  $Z'$  would imply  $0.39 < c'^2 < 0.42$ . At 14 TeV with  $L = 100$  and  $d = 1/2$  the expected 95% constraint on  $Z'$  would imply  $0.40 < c'^2 < 0.41$ . For the heaviest  $Z'$  allowed by the EW fit,  $m_{Z'} = 500$  GeV, the corresponding 95% limits at 7 and 14 TeV would be  $0.37 < c'^2 < 0.43$  and  $0.39 < c'^2 < 0.41$ .

The Tevatron  $Z'$  limits constrain the model for  $m_{Z'} \geq 150$  GeV, corresponding to  $f \gtrsim 920$  GeV, but the region within the 95% EW contour with  $f < 900$  GeV and  $m_{Z'} < 150$  GeV is largely unconstrained. Good EW fits exist down to  $f = 500$  GeV, corresponding to  $m_{Z'} = 85$  GeV. The constraints are relatively weak because LEP II ran sparsely below 150 GeV, accumulating only  $3 \text{ pb}^{-1}$  samples at 130 and 136 GeV. For instance, a 140 GeV  $Z'$  with  $r_{Z'} = 0.02$  corresponding to  $f \simeq 860$  GeV and  $c'^2 = 0.38$  (marked by the ellipse in figure 1) yields a good EW fit to the full data set with  $\chi^2 = 17.5$ ,  $m_H = 600$  GeV, and  $\delta_{FT} = 1.1$ . The resulting shift in  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  at 136 GeV is 0.22 fb, well below the

	Experiment	SM	Pull	LH	Pull
$A_{LR}$	0.1513 (21)	0.1498	0.7	0.1492	1.0
$A_{FB}^l$	0.01714 (95)	0.01683	0.3	0.01670	0.5
$A_{e,\tau}$	0.1465 (33)	0.1498	-1.0	0.1492	-0.8
$\Gamma_Z$	2495.2 (23)	2496.4	-0.5	2496.6	-0.6
$R_\ell$	20.767 (25)	20.743	1.0	20.741	1.0
$\sigma_h$	41.540 (37)	41.480	1.6	41.480	1.6
$R_b$	0.21629 (66)	0.21581	0.7	0.21573	0.9
$R_c$	0.1721 (30)	0.1723	-0.06	0.1723	-0.06
$A_b$	0.923 (20)	0.935	-0.6	0.935	-0.6
$A_c$	0.670 (27)	0.669	0.04	0.668	0.06
$m_W$	80.399 (23)	80.401	-0.07	80.400	-0.02
$A_{PV}$	$-131 (17) \cdot 10^{-9}$	$-159 \cdot 10^{-9}$	1.6	$-157 \cdot 10^{-9}$	1.5
$Q_W(Cs)$	-73.16 (.35)	-73.09	-0.2	-73.23	0.2
$\Delta\alpha^{(5)}(m_Z)$	0.02758 (35)	0.02761	-0.09	0.2754	0.12
$m_t$	173.3 (1.1)	173.3	0.02	173.3	0.02
$\alpha_S(m_Z)$		0.1180		0.1186	
$m_H$		58		560	
$c$				0.14	
$v'$ (GeV)				0.5	
$s_L$				0.077	
$\chi^2/\text{dof}$		8.3/11		8.6/9	
$\text{CL}(\chi^2)$		0.69		0.48	
$m_H[90\%]$ (GeV)		30 — 111		270 — 1000	
$\text{CL}(m_H > 114 \text{ GeV})$		0.04		1	

Table 3: The SM best fit and an LH model fit with  $A_{FB}^b$  and  $A_{FB}^c$  excluded, the LH fit at  $f, c'^2 = 1.47 \text{ TeV}, 0.38$  corresponding to the CDF excess at 240 GeV.

0.67 fb experimental uncertainty.[22] Even at the 95% limit of the EW fit,  $c'^2 = 0.365$ , the effect is only as big as the experimental uncertainty.

For  $f$  approaching 500 GeV the expansion in  $v^2/f^2$  becomes unreliable. Comparing the leading order result for  $\theta_{Z-Z'}$  with the result to all orders[23] we find that the corrections are  $\leq O(10\%)$  for  $f \geq 1$  TeV as naively expected.  $Z - Z'$  mixing is kept under control despite the light  $Z'$  mass, because the factor  $1/5$  that suppresses  $m_{Z'}^2$  is cancelled by a factor  $1 - 2c'^2 \simeq 1/5$  in the off-diagonal matrix element of the  $Z - Z'$  mass matrix. The errors introduced by the leading approximation at the smallest values of  $f$  will shift the values of the parameters at which the best fits occur but will not significantly alter the confidence levels. A quantitatively reliable analysis of the very low  $f$  region will require going beyond the leading approximation.

### Fine-tuning of model parameters

While we have obtained fits to the EW data that resolve the little hierarchy fine-tuning problem, we also find that recent Tevatron data imposes a strong constraint on  $c'$ , the  $U(1)$  mixing angle parameter. As shown in figure 1, the Tevatron data requires  $c'^2$  to be near  $\eta' = 2/5$  to suppress the  $Z'$  coupling to SM fermions (see equation (8)). It is interesting that the EW data requires no further fine-tuning “price”: although simple estimates suggest otherwise, we find that once the current Tevatron constraint on  $c'$  is satisfied, no further tuning is required.

To illustrate the extent of tuning required by the EW data we consider the shift in  $Z$ -fermion couplings from  $Z - Z'$  mixing,  $\delta g_f$ , and the corrections to the effective leptonic weak interaction mixing angle,  $x_W^{\ell,eff}$ , which is the most important pseudo-observable in the EW fit, with part per mil precision,  $\delta x_W^{\ell,eff}/x_W^{\ell,eff} \sim 1 \cdot 10^{-3}$ . The corrections to  $\delta g_f$  from the heavy gauge bosons,  $W'$  and  $Z'$ , are shown in equation (1), while for  $x_W^{\ell,eff}$  they are

$$\delta x_W^{\ell,eff}|_{W',Z'} = \frac{x_W(1-x_W)}{1-2x_W} \frac{v^2}{f^2} \left( -\frac{5}{4} + c^2(1-c^2) + 5c'^2(1-c'^2) \right). \quad (12)$$

The factor 5 amplifying the  $U(1)$  corrections in equations (1) and (12) is especially dangerous.

As a specific example we consider the fit in table 2 of the CDF excess at 240 GeV. The prefactors  $v^2/2f^2$  and  $\simeq v^2/3f^2$  in equations (1) and (12) are then of order 0.01, while the factors in parentheses containing the  $c$  and  $c'$  dependence are generically of order one, suggesting that fine tuning is required. This is indeed the case but the necessary tuning is already imposed by the Tevatron  $Z'$  bounds. Notice first that in equation (1) the  $U(1)$  correction is suppressed not only by the factor  $(c'^2 - 2/5)$  but, as an added bonus, the dangerous factor 5 is offset by the factor  $(1 - 2c'^2)$  which is  $\simeq 1/5$  if  $c'^2 \simeq 2/5$ . The  $SU(2)$  correction is also small, as the EW fit prefers small values for  $c$ , typically between 0.1 and 0.3, with for instance  $c = 0.24$  for the fit in table 2. The net result for the correction to

the  $Zee$  coupling is then  $\delta g_e \simeq 2 \cdot 10^{-4}$ , two orders of magnitude below the naive estimate and within the range of the experimental uncertainty. A similar miracle occurs for  $\delta x_W^{\ell, eff}$ , equation (12), where the term  $-5/4$  is offset by the  $c'$  dependent term, (e.g., at  $c'^2 = 2/5$  we have  $-5/4 + c'^2(1 - c'^2) = -1/20$ ) with some further reduction provided by the  $SU(2)$  term. The net result,  $\delta x_W^{\ell, eff} \simeq 1.7 \cdot 10^{-4}$  is again reduced by two orders of magnitude from the naive estimate and falls within the precision of the measurement. The suppression of these (and the other) EW corrections is assured just by the value of  $c'$  imposed by the Tevatron data with no additional fine tuning.

Corrections from the  $T$  parameter due to the top partner,  $t'$ , and the triplet Higgs vacuum expectation value,  $v'$ , can also have an appreciable effect on the fit, but they are not fine-tuned and they are not solely or even primarily responsible for the preference for large values of  $m_H$  in the fits. For instance, for  $f$  and  $c'$  fixed by CDF data as in table 2, there is an acceptable fit with  $T_{t'} = T_{v'} = 0$ , falling within the 95% CL contour of figure 1, with  $\chi^2 = 20.1$ ,  $\delta_{FT} = 0.11$ , and  $m_H = 700$  GeV. Allowing  $T_{t'} = 0.11$  and  $T_{v'} = 0.01$ , the fit in table 2 improves to  $\chi^2 = 17.7$ ,  $\delta_{FT} = 0.9$  at  $m_H = 820$  GeV. These values are not fine-tuned, as a broad range of other values of  $T_{t'}$  and  $T_{v'}$  also provide robust (and in some cases slightly better) fits, with a wide range of  $m_H$  values, between 300 GeV and 1 TeV. The values of  $T_{t'}$ ,  $T_{v'}$ , and  $m_H$  are indeed correlated but they are not fine tuned. Different values of  $T_{t'}$  and  $T_{v'}$  can be accommodated with different choices of  $m_H$ . The correlations imply predictions that will be tested if evidence for the model emerges and the model parameters are measured.

## Discussion

Contrary to earlier studies we find that the original littlest Higgs model with universal fermion couplings is consistent with precision EW data while ameliorating the little hierarchy problem, as originally envisioned in [1]. Our conclusions differ from the earlier studies[3, 5] chiefly because we have scanned over all SM and LH parameters, as might not have even been possible for the earlier studies with the computing capability available at the time. However, in the intervening years the Tevatron limits on  $Z'$  production have increased to the point that they now constrain the model more strongly than the EW data. The excess observed by CDF at 240 GeV is consistent with the predicted  $Z'$  boson; if it is confirmed the LH model will be one of the possible explanations. If it is not confirmed and no other consistent signal is seen at the Tevatron or the LHC, the model will succumb to a fine tuning problem for the  $U(1)$  mixing angle  $c'$  that is as severe as the tuning required by the little hierarchy problem that it purports to solve.

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